The Worldwide Center of Mathematics

On umbilic points

Farid Tari

ICMC, USP, Brazil

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Talk: 4-5pm
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Abstract: There are various equivalent ways to define umbilic points on smooth and regular surfaces in the Euclidean 3-space. They are points where the shape operator is a multiple of the identity, points where the principal curvatures are equal, points where the first and second fundamental forms are proportional, or singular points of the binary differential equation of the lines of principal curvatures.
The Carathéodory conjecture states that any convex and closed surface has at least 2 umbilic points. That there is at least one follows by Poincaré-Hopf Theorem applied to the direction field determined by one of the principal directions. That there are at least 2 is, to my knowledge, a still open problem for smooth surfaces in the Euclidean space (since 1924).

In this talk, I will recall the result where I showed that any convex and closed surface in the Minkowski 3-space has at least 2 umbilic points. I will also speak about a recent work with Masaru Hasegawa on umbilic points on newly born surfaces.The simplest way to have birth of surfaces is through transitions in the fibres of a smooth function f:R^3 –> R with a Morse singularity of index 0 or 3 at some given point. (For example, take f(x,y,z)=a^2x^2+b^2y^2+c^2z^2, abc≠0, and consider the set f(x,y,z)=t, with t varying near zero. When t<0, we get the empty set; at t=0, we get a point and, for t>0, we get an ellipsoid.) We show that newly born surfaces in the Euclidean 3-space have exactly 4 umbilic points (and all of them have the so-called type lemon), provided that the Hessian of f at the singular point has pairwise distinct eigenvalues. When only two of such eigenvalues are equal, the number of umbilic points is either 2, 4, 6 or 8. The same results holds for newly born surfaces in the Minkowski 3-space. In that case, when the two eigenvalues associated to the two spacelike eigenvectors are distinct, we get exactly 4 umbilic points all of type lemon. If they are equal, the number of umbilic points is either 2, 4, 6 or 8.

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